

February 9, 1888.

Dr. E. FRANKLAND, Vice-President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read:—

- I. "The Small Free Vibrations and Deformation of a Thin Elastic Shell." By A. E. H. LOVE, B.A., Fellow of St. John's College, Cambridge. Received January 19, 1888.

(Abstract.)

In this paper the method employed by Kirchhoff and Clebsch for the treatment of a thin plane plate is applied to the case of a thin shell, or plate of finite curvature. The form of the potential-energy-function for the strain in an element of the shell is the same as that obtained by Kirchhoff for a plate, the quantities depending on the curvature of the surface being replaced by the difference of their values in the strained and unstrained states. It is proved that only for an inextensible spherical surface is this function the same function of the changes of principal curvature as for a plane plate. The general equations of equilibrium and small motion under the action of any system of forces are formed. It is shown that in general the shell cannot vibrate in such a manner that no line on the middle-surface is altered in length, because this condition makes it impossible to satisfy the boundary conditions which hold at a free edge. It then appears that approximate equations of motion may be taken, in which the terms of the potential energy depending on the bending may be neglected, and only those depending on the stretching need be retained. It is shown that surfaces of uniform curvature with no bounding edges are the only ones which admit of purely normal vibrations, and that vibrations in which the displacement is purely tangential are possible on all shells whose middle-surfaces are surfaces of revolution bounded by small circles. The cases of the spherical and cylindrical shell receive special discussion. The equations of motion can always be solved, but solutions of the frequency equations could only be obtained in case the displacement was symmetrical about the axis. The application of the general equations to problems of equilibrium is illustrated in the case of spherical shells, for which

the equations can be solved; some special examples are given. For the sake of greater intelligibility, I have included an historical account of previous theories of plane plates and shells, a description of the method of the present paper, and a summary recapitulating the chief physical results.

II. "True Teeth in the young *Ornithorhynchus paradoxus*." By EDWARD B. POULTON, M.A., F.L.S., of Jesus and Keble Colleges, Oxford. Communicated by W. K. PARKER, F.R.S. Received January 26, 1888.

For the purpose of continuing some recent work upon various epidermic structures in *Ornithorhynchus*, Dr. Parker very kindly placed his most valuable material at my disposal. Among other things was a series of consecutive vertical transverse sections through the head of a young individual, about 8·3 decimetres long, when in the curled-up attitude in which it had been received, and which was fixed by the spirit. In this specimen only the larger hairs had appeared above the surface of the skin.

The sections had been prepared for Dr. Parker by his son, Professor W. Newton Parker, of Cardiff, and although intended for the investigation of morphological points in connexion with the development and structure of the skull, many of them were in every way adapted for minute histological investigation. Examining these sections I found that large and apparently typical mammalian teeth were developing in the subepithelial tissues on each side of the roof of the mouth. I at once communicated with Dr. Parker, telling him of the discovery, and enquiring whether he had any objection to the publication of the fact. Dr. Parker replied, and urged me to at once communicate the discovery to the Royal Society, at the same time offering me material in the most free and generous manner for the further investigation of the dental structures in *Ornithorhynchus* and in *Echidna* (if present in the latter). When it is remembered that Dr. Parker had put the sections aside for a time in consequence of the press of other work, intending soon to make use of them for the investigation of the skull, it will be seen at once that my association with this discovery is purely accidental, and that I have been treated in an extremely generous spirit.

As the lower jaw was not included in the sections, I cannot yet state that teeth are present in it, but there is little doubt that this is the case.* Teeth were present in the upper jaw, in thirty sections through the head, and of these all, except the nine anterior sections, included some part of the eye. The teeth probably represent some

* I have since found that teeth are present in the lower jaw.—Feb. 6, 1888.